

Considering variability and uncertainty

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A common issue among workers measuring soil water content is how many measurements are required for a good estimate of water content in a managed unit. A unit can be a small test plot, an irrigated field, or a waste storage site. Multiple measurements taken within a meter of each other often show a surprising amount of variability. The uncertainty introduced by the spatial variability can render a water content estimate useless.

Uncertainty from water content variability adds to other uncertainties inherently present in each measurement. Most significant is uncertainty from instrument error, which is any distortion of the actual value introduced by the sensor, including calibration error. While the uncertainty from instrument error is beyond control of the user, careful sampling and prudent interpretation of measured values will improve the water content estimate when spatial variability is present. This article presents a simple approach for improving water content estimation under spatially variable conditions.

Spatial Variability

A measurement at a single location is seldom representative of water content in the entire unit. Variability of soil water content can result from spatial differences in soil parent material, plant transpiration, erosion, compaction and other processes that affect soil structure.

Fundamentally, the scale of an individual measurement must be considered relative to the scale of the spatial variability. Variability can be present on scales of centimeters to kilometers. If an instrument has a sensitive volume large enough to sample an entire unit with a single measurement, spatial variability does not affect the estimate. But the sensitive volume of available water content measurement methods is small enough that spatial variability must be considered when choosing a measurement scheme. Given this difference in scale, measurements at multiple locations are required when estimating water content in a unit.

Relating Spatial Variability and Uncertainty

One way to express the results of multiple water content measurements is to combine the mean with the uncertainty of the mean. An example data set is used to

demonstrate this method. Ten water content measurements were collected with a HydroSense™ (see article on adjacent page) at different locations in a cropped field.

The seven percent difference between

Units: m ³ m ⁻³				
0.18	0.18	0.24	0.17	0.17
0.22	0.18	0.22	0.22	0.22
mean = 0.20				
standard deviation = 0.025				
range = 0.07				

the high and low readings is evidence of spatial variability and illustrates that a single measurement is inadequate. The mean value is accepted as the best estimate. The uncertainty of the mean volumetric water content ($\delta\theta_v$) is calculated by

dividing the standard deviation (s) by the square root of the number of measurements (n). The water content estimate with uncertainty (θ_v^*) for this data set is with $\bar{\theta}_v$ the mean of

$$\theta_v^* = \bar{\theta}_v \pm \delta\theta_v = \bar{\theta}_v \pm \frac{s}{\sqrt{n}} = 0.20 \pm 0.008 m^3 m^{-3}$$

individual measurements. Assuming normal distribution of the data, this result states that the mean of the ten measured values is within ± 0.008 of the actual water content value with a confidence level of 68 percent. The confidence level increases to 95 percent if the uncertainty is doubled;

$\theta_v^* = 0.20 \pm 0.016 m^3 m^{-3}$. To add perspective to this, let the unit be the size of a football field with a soil profile depth of 20 cm. The uncertainty of $0.008 m^3 m^{-3}$ or 0.8 percent volumetric water content translates into nearly 2000 gallons of water. Compare this with the uncertainty of nearly 6000 gallons for a single measurement.

Combining Measurements

There are instances when two measurements must be added or multiplied to calculate an estimate. In general, if two quantities are added to calculate an estimate, the uncertainties of the individual quantities are added to get the total uncer-

tainty. When the quantity is a product or quotient, *relative* uncertainties are added. The relative uncertainty is simply the uncertainty divided by the mean.

Relative uncertainty is applied to a sample data set collected to estimate volumetric water content using oven drying and known sample volume.

Assuming the density of water is 1000

	Gravimetric water content (kg kg ⁻¹)	bulk density (kg m ⁻³)
mean (\bar{x})	0.199	1233
standard deviation (s)	0.006	74.0
uncertainty (δx)	0.002	23.4
relative uncertainty ($\frac{\delta x}{\bar{x}}$)	0.009	0.019

kg m⁻³, the mean volumetric water content is the product of the mean gravimetric water content ($\bar{\theta}_g$) and the mean bulk density ($\bar{\rho}_b$) and is $0.24 m^3 m^{-3}$ for these data. The uncertainty is expressed with the mean in the estimate as

$$\theta_v^* = \bar{\theta}_v \pm \left[\frac{\delta\theta_g}{\bar{\theta}_g} + \frac{\delta\rho_b}{\bar{\rho}_b} \right] * \bar{\theta}_v = 0.24 \pm 0.007 m^3 m^{-3}$$

This result states that the actual volumetric water content lies between 0.233 and 0.247 at a 68 percent confidence level or between 0.226 and 0.254 at a 95 percent confidence level. The uncertainty for bulk density is generally higher because of errors from core sampling.

Summary

Spatial variability of soil water content must be considered when measurements are collected to estimate the water content of a managed unit. The uncertainty of the estimate can be calculated using simple statistical methods. The calculation of uncertainty can then provide guidance on the number of measurements required to obtain the desired level of confidence in the estimate. Experience in monitoring water content and careful evaluation of measurements will lead to an optimum sampling scheme and efficient use of water.